Time diffraction of evanescent waves

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The concept of time diffraction of evanescent electromagnetic waves is proposed. Transient propagation of electromagnetic waves is studied in two forms: propagating and evanescent waves. Differences and similarities between quantum particle tunneling and photon tunneling are clearly demonstrated. Traversal time cannot be accurately defined due to diffraction in time. Nevertheless, a delay time is defined as the difference between the time of flight and the peak time of the transient light. The delay time is found to increase linearly and slowly with the traversal distance. Superluminal tunneling is found possible only for evanescent waves, and Einstein causality is not violated. [S1063-651X(99)12211-6]

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Propagation of electromagnetic waves in one dimension can be complicated when the waves travel through an evanescent area. The evanescent area can be, for example, a metallic barrier [1] or a photonic band-gap structure [2,3]. Despite the exponential decay, a small part of the waves may tunnel through the area, a situation closely analogous to the one-dimensional tunneling of a quantum particle through a potential barrier [1,4].

It is interesting to note that as one of the few solvable problems in quantum mechanics, one-dimensional tunneling of a monochromatic wave function through various classically forbidden potential barriers has been included in every basic textbook in quantum mechanics. This is to demonstrate the nonclassical concept of spatial distribution of probability of a quantum particle. There appears no difficulty to analytically obtain the stationary field distribution in the propagating as well as in the evanescent regions, and the transmission and reflection coefficients can be accurately determined. However, despite the clear picture, the subject of tunneling has for a long time been controversial (see, e.g., a review in Ref. [5]). What have been in dispute are not the stationary states in textbooks, but rather the transient terms, for example the time it takes to tunnel through a barrier. So far, various times have been proposed as a measure of the tunneling time. The standard treatment for waves is to look into the collective movement of a group of waves, and to define a group velocity and a phase velocity. In one-dimensional tunneling, based on the stationary states, the phase time for the tunneling can be readily formulated [5,1]. Other proposed times include the dwell time that averages a beam of particles, the interaction time [6] that concerns time-modulated potentials, and the Larmor time [7] that uses a local time reference. An important reason for proposing that many time scales stems from the theoretical prediction [8,9] and experimental confirmation [2,3,10] of the existence of superluminal tunneling, i.e., a short light pulse travels faster by tunneling through a barrier than by moving in vacuum, an apparent violation of the Einstein causality. In earlier times, the cases where the group velocity exceed the light speed in vacuum were believed to be of no appreciable physical significance [11]. Recent confirmative experiments [2,3] brought back the reality of the superluminal tunneling. Facing this embarrassing situation, many attempted to explain the phenomenon as a reshaping of the pulse [2,5], i.e., attenuation of the pulse in the barrier shifts the peak of the pulse forward. But little is known why the barriers attenuate pulses unevenly.

So far, it seems we are confident to discuss the tunneling with a mixed terminology of photons and quantum particles. It is also widely believed that experiments with photons can be directly used to understand particle tunneling. This understanding originates from the apparent resemblance of the two cases in terms of the stationary states [1,4]. In both cases the waves satisfy the same form of time-independent Helmholtz equation inside and outside the barrier area. However, one should bear in mind that as far as time-dependent tunneling is concerned, the transient terms are the only things that matter, and to decompose the time-dependent part from the wave functions is no longer trivial.

In the present paper, we shall study the transient terms in photon tunneling, together with a close comparison with the particle tunneling. We note that the diffraction in time for quantum particles in one dimension was studied several decades ago [12]. To make a connection, we shall recapitulate some important results from Ref. [12] and we shall then present new results on time diffraction of photons. A numerical example of a metallic barrier will be presented to demonstrate the diffraction. After the numerical example, the conclusions will be presented.

In 1952, Moshinsky [12] studied the shutter problem as depicted schematically in Fig. 1, where a monochromatic beam of noninteracting particles of mass m and energy $\hbar^2k^2/2m$ travels along the x axis from the left to the right, at $x=0$ the beam is stopped by a shutter of a perfect absorber which is perpendicular to the beam, and at $t=0$ the shutter is opened. A transient current of the particles can be observed at a time $t>0$ and a distance x from the shutter. Mathematically, the shutter problem can be described as follows. The initial wave function is

$$\psi(x,t=0)=\Theta(-x)e^{ikx}, \tag{1}$$

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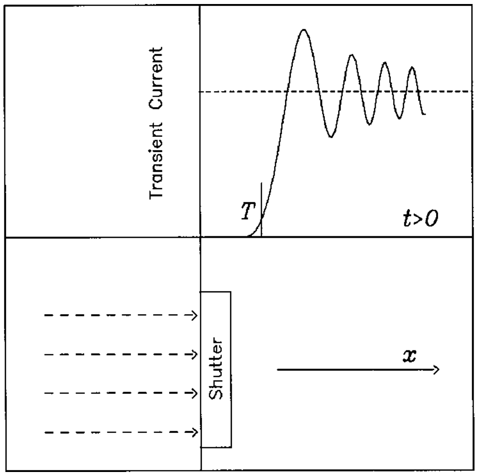

FIG. 1. The Moshinsky shutter.

where $\Theta(-x)$ is the Heaviside unit step function: $\Theta(<0)=0$ and $\Theta(\geqslant 0)=1$. The wave function after the shutter is opened $(t>0)$ satisfies the time-dependent Schrödinger equation:

$$\frac{\partial^2\psi(x,t)}{\partial x^2}=\frac{-2mi}{\hbar}\frac{\partial\psi(x,t)}{\partial t}. \tag{2}$$

A solution of Eqs. (1) and (2) is the Moshinski function [12]:

$$\mathcal{M}(x,k,t)=\frac{1}{2}e^{i(kx-\hbar k^2 t/2m)}\mathrm{erfc}[(x-\hbar kt/m)/\sqrt{2i\hbar t/m}], \tag{3}$$

where erfc(z) is the complementary error function of a complex variable. The transient current can then be calculated by

$$J(x,t)=\frac{\hbar}{2im}\left[\mathcal{M}^*(x,t)\frac{\partial\mathcal{M}(x,t)}{\partial x}-\mathcal{M}(x,t)\frac{\partial\mathcal{M}^*(x,t)}{\partial x}\right]. \tag{4}$$

A typical time dependence of the current at a fixed distance x is shown also in Fig. 1, where the current is normalized to the stationary velocity $v=\hbar k/m$ and T is the time of flight, $T=x/v$. After a sufficient time $t\sim\infty$, the transient current becomes stable at the stationary current $J(x,t\sim\infty)=v$ (the dotted line in Fig. 1). It is interesting to note that the curve in Fig. 1 has the familiar form of Fresnel diffraction of light in semi-infinite space (see p. 430 in Ref. [11]). The Fresnel diffraction is in the spatial domain while the diffraction shown in Fig. 1 is in the time domain. We make two remarks on the time diffraction in Fig. 1. (i) This temporal diffraction effect was recently verified experimentally with atomic waves [13]. (ii) One can see from Fig. 1 that, before the time of flight, the observer can already see a flow of particles, which implies that nonrelativistic particles may travel faster than the time of flight. However, Moshinsky showed in Ref. [12] that there was no similar transient solution for the Klein-Gordon equation, which is the equation that relativistic particles obey. In the relativistic case, the transient current remains exactly zero before the time of flight [12]. Therefore, superluminal travel is not possible for relativistic particles, and the diffraction in Fig. 1 can only be true for slow particles.

Having discussed the Moshinsky shutter problem, we turn our attention to the case of photons. For the sake of an easy terminology, we shall specify our evanescent area to be a metallic barrier. Therefore, in the case where free current and charge are both absent (i.e., $\vec{\nabla}\cdot\vec{E}=0$ and $\vec{\nabla}\times\vec{B}=\vec{0}$), one obtains from the Maxwell equations the well-known wave equation:

$$\frac{\partial^2 E(x,t)}{\partial x^2}=\mu\epsilon\frac{\partial^2 E(x,t)}{\partial t^2}+\mu\sigma\frac{\partial E(x,t)}{\partial t}, \tag{5}$$

where μ is the permeability, ϵ is the permitivity, and σ is the conductivity. In the following, let us consider separately two extreme cases, $\sigma\sim 0$ and $\sigma\sim\infty$.

In the case of a dielectric medium $(\sigma\sim 0)$, one has from Eq. (5)

$$\frac{\partial^2 E(x,t)}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 E(x,t)}{\partial t^2}, \tag{6}$$

where, to simplify the notation, we have assumed the vacuum light speed $c=1/\sqrt{\mu\epsilon}$ with the approximation $\mu\sim\mu_0$ and $\epsilon\sim\epsilon_0$, which will not effect the generality of our discussions. The transient solution of the above equation under the Moshinsky shutter initial condition in Eq. (1) can be easily obtained as [12]

$$E(x,t)=\Theta(t-x/c)\left[e^{ik(x-ct)}-\frac{1}{2}\right]. \tag{7}$$

One realizes that, for propagating waves, the transient field jumps suddenly from zero to $\frac{1}{2}$ at the time of flight $t=x/c$, which confirms that superluminal traveling cannot be possible for propagating waves.

The more interesting case is $\sigma\sim\infty$, a perfect conductor. In this case, one only keeps the second term on the right side of the wave equation in Eq. (5) and writes down the wave equation as

$$\frac{\partial^2 E(x,t)}{\partial x^2}=\frac{\sigma}{\epsilon_0 c^2}\frac{\partial E(x,t)}{\partial t}, \tag{8}$$

where we have also assumed $\mu\sim\mu_0$. Comparing the above evanescent wave equation to the time-dependent Schrödinger in Eq. (2), one immediately writes down the transient solution of Eq. (8) under the Moshinsky shutter initial condition in Eq. (1) as

$$\mathcal{X}(x,k,t)=\frac{1}{2}e^{ikx}e^{-c'k\omega t}\mathrm{erfc}[(x+2ic'\omega t)/\sqrt{4c'ct}], \tag{9}$$

where $c'\equiv c\epsilon_0/\sigma$ and $\omega=ck$ is the light angular frequency.

Comparing the two solutions in Eqs. (3) and (9), one realizes that the main difference between the two functions is that $\mathcal{M}(x,k,t)$ has a time-dependent oscillation $e^{i\hbar k^2 t/2m}$, whereas $\mathcal{X}(x,k,t)$ contains a time-dependent exponential de-

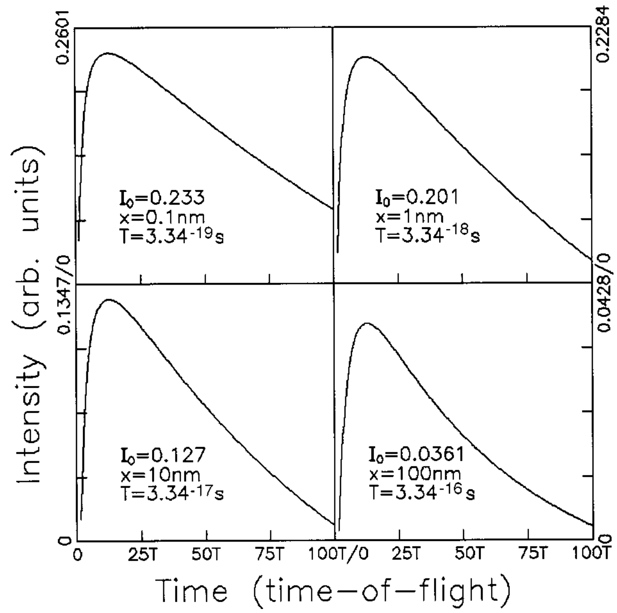

FIG. 2. Time diffraction of evanescent waves for four traversal distances x. The time scale (horizontal axis) is $t=0\sim 100T$ for each curve. I_0 is the peak intensity (perpendicular axis).

cay $e^{-c' k \omega t}$. Furthermore, for sufficient long times $t\sim\infty$, at any distance x, the waves $\mathcal{X}(x,k,t\sim\infty)$ reduce to zero. So, what one may expect from the function $\mathcal{X}(x,k,t)$ is a time diffraction similar to the Moshinsky function $\mathcal{M}(x,k,t)$, but the new diffraction will not have the time-dependent oscillation and will reduce to zero at stationary states. It is worth noting that in realistic cases one would expect both oscillation and exponential decay, but at least in the case of a metal the exponential decay dominates. In the following, let us present a numerical example of the intensity flow of the evanescent waves,

$$I(x,t)=\mathcal{X}(x,k,t)\mathcal{X}^*(x,k,t), \qquad (10)$$

where the conductivity σ is calculated from the plasmon frequency ω_p and damping rate γ of the metal, as

$$\sigma(\omega)=\frac{i\epsilon_0\omega_p^2}{\omega+i\gamma}. \qquad (11)$$

We choose silver to be the metal and we assume the light wavelength $\lambda=685$ nm [1–3]. Other parameters are chosen from Ref. [1], $\hbar\omega_p=9.04$ eV and $\gamma=0.002\omega_p$.

In order to show the expected time-diffraction spot, we present in Fig. 2 four calculated curves for $x=0.1,1,10$, and 100 nm, respectively. The time is scaled to the time of flight in vacuum $T=x/c$, and the time scale for each curve is $0\sim t\sim 100T$. For each curve one finds a clear diffraction spot. The forward and backward tails of the spot tend to zero, and the spot peak occurs sometime after T. Interpretation of Fig. 2 is as follows. For an observer at a distance x, there is a certain time interval when the observer may see the light, and the intensity of the observed light is a function of time. Apparently, it may also state that at a given time, the observer receives changing light at different distances. One also concludes that superluminal tunneling through an evanescent

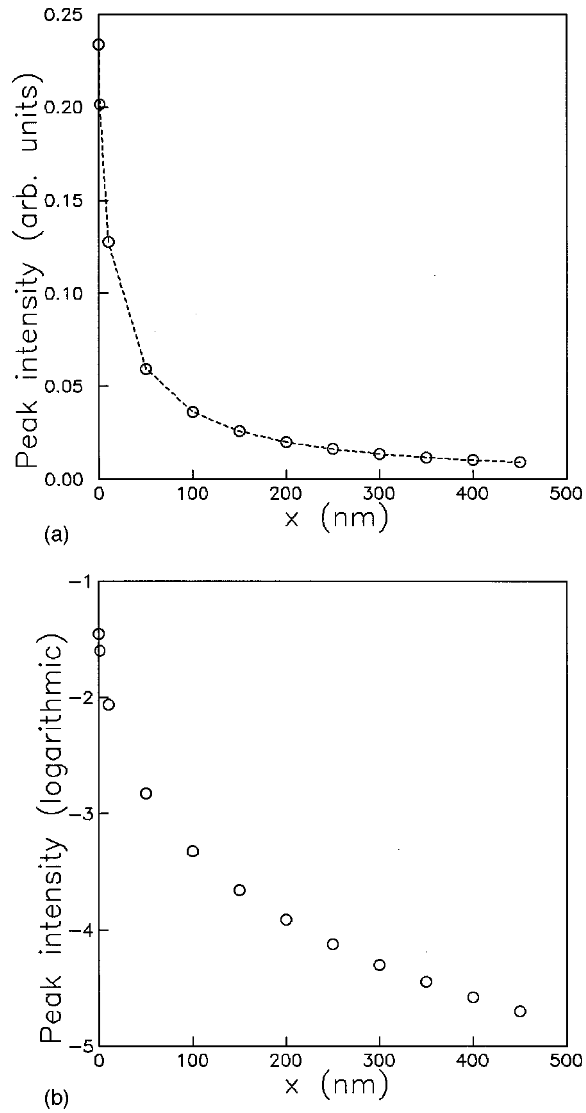

FIG. 3. Distance dependence of peak light intensity in direct (a) and logarithmic (b) scales.

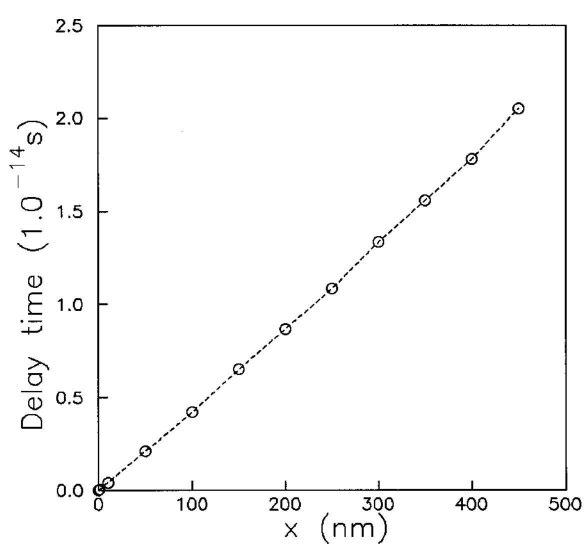

FIG. 4. Distance dependence of delay time.

region is possible for photons, due to the fact that the left tail of the time spot extends to the times shorter than T.

The calculation has been extended for more distances 0.1 nm$\sim x \sim$450 nm in Figs. 3 and 4. Figure 3(a) shows the distance dependence of the peak light intensity of the time-diffraction spots, which, as one may expect, exhibits a rapid decay. In order to have a closer look at the decay, we present the same curve in a logarithmic scale in Fig. 3(b), where one realizes that after the first three points the rest of the curve is exponential [a straight line in the logarithmic plot in Fig. 3(b)], and the first three points decay even more rapidly. As far as traversal time is concerned, we are still in an embarrassing situation where one does not know how to define an accurate time scale because of the time diffraction. However, at least we can define a delay time which is the difference between the time of flight and the peak time of the time-diffraction spot. We have found an interesting phenomenon that the delay time is almost constant for different distances, which seems consistent with an earlier prediction [14] and with the recent experimental results that showed a lack of thickness dependence of the tunneling time [3]. A closer look at the delay time reveals that the delay time increases slowly and linearly with the increasing distances as shown in Fig. 4. One learns from Fig. 4 that the delay time increases considerably slowly from about 0.05 to 2.1×10^{-14} s within the same time scale while the distance increases from 1 to 450 nm. This means that increasing the thickness of the evanescent area results in small changes in the delay time. If the difference cannot be resolved in experiments, the delay time might be considered constant.

In passing, we follow other authors on this subject to show our loyalty to the principle of the Einstein causality. We assume that there is a waiting process for the arriving waves to establish themselves in front of the barrier. When the waves are establishing, a forward tail of the waves is already extended into the barrier. Although, for every part of the waves the traversal distance is the same, but, for a certain observation time, the forward part may be closer to its peak light intensity than the later parts, or, equivalently, an earlier part of an incident wave train may suffer less attenuation. Therefore, the peak of an incident light pulse is shifted forward, so that its group velocity exceeds the light speed in vacuum. The above discussion details the previous conjecture of the reshaping process.

Let us now summarize the present work. We have proposed the concept of time diffraction of evanescent light waves, which is shown to be useful to clarify the one-dimensional tunneling of photons. We have also shown the difference and resemblance between particle and photon tunneling. According to Ref. [12], transit propagation of nonrelativistic particles that obey the time-dependent Schrödinger equation results in Fresnel diffraction in the time domain, whereas the propagation of relativistic particles that obey the Klein-Gordon equation cannot show time diffraction. In the present work, we have found that the evanescent electromagnetic waves are also subject to time diffraction, while propagating electromagnetic waves are not. Based on the discovered time diffraction, we have concluded that an accurate traversal time cannot be defined. We have demonstrated that the delay time, which is the time interval between the time of flight and the peak time, is a slow and linear function of the traversal distance. Similar to the case of nonrelativistic particle tunneling, we have shown that superluminal tunneling is possible for evanescent waves. We have argued with a discussion of the reshaping process that the Einstein causality is not violated in the cases where the group velocity of a light pulse exceeds the light speed.

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